

Illinois Real Estate Letter

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How Real Estate Brokers Affect the Outcome of Negotiations Between Buyers and Sellers

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Real estate markets can best be described as having imperfect information; buyers and sellers do not know the magnitudes of each others' *reservation prices*. This lack of information motivates *search activity*, with buyers and sellers seeking trading partners. Once a seller meets a buyer who is interested in the seller's property, the two sides engage in negotiations to determine the transaction price.

Intermediaries play a crucial role in a variety of markets with imperfect information. For example, in real estate transactions, brokers assist in both the search process and the negotiation stage. They also provide assistance to the buyer and seller in the listing, pricing, contracting, financing, and insurance stages of the transaction. The significance of the role played by brokers is evident by the fact that, in most housing markets, they assist with the sale of more than 80% of single-family dwellings. It is not surprising, therefore, that a large body of research has been devoted to studying the role of real estate brokerage.

The academic literature has focused on various aspects of the brokerage profession. A number of studies have investigated whether too many or too few resources are allocated to brokerage services. Another line of research has examined how alternative commission

structures (percentage commission, flat fee commission, and net listing) affect the transaction price and the broker's incentives to expend the optimal amount of effort for his or her client. Recently, a number of studies have looked at the emergence of buyer brokerage and its implications for the future of the industry.

The question that has attracted the most attention, however, is how the prices of houses sold through brokers compare to the prices of those sold directly by the owners. The empirical (based on data from actual transactions) studies on this issue have produced mixed results. While some studies have reported that the price would be higher if a broker were employed, others have found no significant difference in prices. There are even studies that have pointed to higher prices for FSBO (For Sale By Owner) sales compared to brokered sales.

Because there are a number of unobservable factors that influence the sale price of a property, it is not surprising that the empirical literature has produced mixed results on brokerage's impact on sale prices. In addition to being a function of physical and locational characteristics of the property, the sale price is affected by the negotiation process between the buyer and seller, and by the way the negotiation process is

impacted by the broker. But, as noted, many of the factors that play roles in the negotiation process are unobservable. We do not, for instance, have data on the buyer's or seller's sense of urgency in completing the transaction, or their reservation prices and their bargaining powers.¹ Neither do we have available data on the offers and counter-offers made by the two parties prior to the agreement, and how these offers correlate with the final negotiated price.

One way to get such information is to contact home buyers and sellers and survey them, but this method can be very costly, and is often unreliable. A more efficient and reliable alternative is to derive the data through *laboratory experiments*. Under this methodology, a number of subjects (mostly students) are recruited to play the roles of different players in the market. The experiments are set up in such a way that they represent the market under consideration, and the subjects are given monetary incentives to act in a profit maximizing manner: they are rewarded based on their performance in the experiments. This method has been widely used in economics, finance, psychology, accounting, marketing, and other social sciences.

In this article, we report the results of a set of experiments that we conducted to analyze the role of brokers in bargaining between a buyer and a seller.² The purpose of the experiments was to study how the presence of a broker affects the transaction price, the likelihood of a successful negotiation, and the time it takes until a negotiation is completed.

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We examined these issues under two different informational structures. In one case, the broker was “uninformed,” in that he or she had no information about the seller’s and buyer’s reservation prices beyond what was already common knowledge. In the other case, the broker was “informed,” in that he or she had more information about the buyer’s reservation price than the seller did, and had more information about the seller’s reservation price than the buyer did. These two information structures allowed us to examine whether brokers with superior information can use their advantage to improve the efficiency of bargaining.

The Experimental Design

We conducted three experimental sessions at the Pennsylvania State University. In the first session, buyers and sellers bargained with each other directly. In the second and third sessions, a buyer and seller bargained with each other through a broker. Subjects’ cash earnings in the experiment depended on the outcomes of their negotiations. Thus, each subject had genuine incentives to negotiate the best deal that he or she could.

The subjects negotiated with each other through computers. Special software was designed to facilitate the bargaining among the subjects. Each session consisted of 25 periods, with the subjects negotiating a new deal in each period. To simulate the fact that a home buyer and a home seller participate in a limited number of house transactions during their lifetimes, the computer program was designed to match each buyer with a new seller and broker in each period.

There were twelve subjects in the first session, who were divided into two groups: six buyers and six sellers. In each of the 25 periods, each seller had a unit to sell and each buyer wanted to buy a unit. The seller and the buyer in each pair negotiated with each other as follows. First, the seller posted a price for his or her unit. The buyer either accepted this price or rejected it and made a counter-offer. Then, the seller either accepted the buyer’s offer, or rejected it and made another counter-offer, and so on.

The players were allowed to send messages to each other along with their price offers and counter-offers. When

either the buyer or the seller accepted the other’s offer, the negotiation between that pair ended for that period. Each participant then went on to the next period, during which he or she was paired with a new negotiating partner. The subjects were given five minutes in each period to reach an agreement.

The sellers’ and buyers’ reservation prices were randomly assigned by the computer in each period. Each seller could see his or her reservation price for that period on a computer screen, but did not know the buyer’s reservation price. Similarly, each buyer could verify his or her reservation price for that period, but did not know the seller’s reservation price. However, all subjects were informed of the *range* of possible values for the seller and buyer reservation prices. The range of possible values for the seller’s reservation price was \$601 to \$700; the range for the buyer’s reservation price was \$751 to \$850.

If the seller and buyer agreed on a price during a period, then the seller’s earnings equalled the negotiated price minus the seller’s reservation price, while the buyer’s earnings equalled the buyer’s reservation price minus the negotiated price. For example, if a seller with a \$690 reservation price and a buyer with a \$770 reservation price settled on a \$750 sale price, the seller would earn $(\$750 - \$690) = \$60$ and the buyer would earn $(\$770 - \$750) = \$20$. If the two had failed to agree on a price in a particular period, then the seller and buyer each would have earned zero for that period.

The second session was similar to the first, except there were eighteen subjects: six buyers, six sellers, and six brokers. In each period, each buyer was randomly matched with a seller and a broker. All negotiations between the buyer and seller went through the broker. The broker did not know the seller’s or the buyer’s reservation price, functioning solely as a “middleman” whose role was to facilitate bargaining by conveying messages between the seller and buyer.

The bargaining process proceeded as follows. First, the seller consulted with the broker, and instructed the broker to post a price for his/her unit. The broker conveyed the seller’s offer to the buyer, along with an optional recommendation

as to whether to accept or reject it. The buyer either accepted this price, or else rejected it and sent a counter-offer, along with an optional message to the broker. The broker then conveyed the buyer's offer to the seller, again with an optional recommendation. The seller, in turn, either accepted the buyer's offer, or else rejected it and made another offer through the broker, and so on.

When either the buyer or the seller accepted the other's offer, the negotiation for that period ended. If the seller and the buyer agreed on a price, their earnings were computed as above, except that the seller had to pay the broker 6% of the negotiated price as a commission. If the seller and buyer failed to agree on a price in a particular period, then the seller, the buyer, and the broker each received zero in earnings for that period.

The third session was identical to the second, with one exception: the broker was endowed with additional information about the reservation prices of the buyer and seller with whom he or she was matched. Specifically, the broker knew the quartiles of the ranges into which the seller's and buyer's reservation prices fell. Thus, while the broker did not have full information, he/she had more specific information about the seller's reservation price than the buyer did, and more specific information about the buyer's reservation price than the seller did.

Results of the Experiments

We first present results concerning the broker's effect on the negotiated price. Since sellers incorporate commission expenses into their reservation prices, and since higher reservation prices for sellers would lead to higher sale prices, we had expected sale prices to be higher with brokers than without them. In other words, we expected a seller to shift some of the commission expense to the buyer.

On the other hand, we did not expect informed brokers to have a different effect on prices than uninformed brokers would. While it may seem that an informational advantage should help the broker convince a buyer to agree to a higher price, the buyer has no reason to believe the broker's recommendation,

than without one. The average price was also slightly higher in the session with an uninformed broker than in the session with an informed broker. When we calculated the average fraction of the broker's fees that were shifted to buyers, we found that the seller shifted 79% of the commission to the buyer when the broker was uninformed, but only 67.9% when the broker was informed.

As we had expected, an increase in the seller's or buyer's reservation price led to a higher negotiated price. We also found, in all three sessions, that the agreed-on price is positively related to both the seller's initial asking price and the buyer's initial offer. This result indicates that the seller's initial asking price signals the buyer about the lowest price

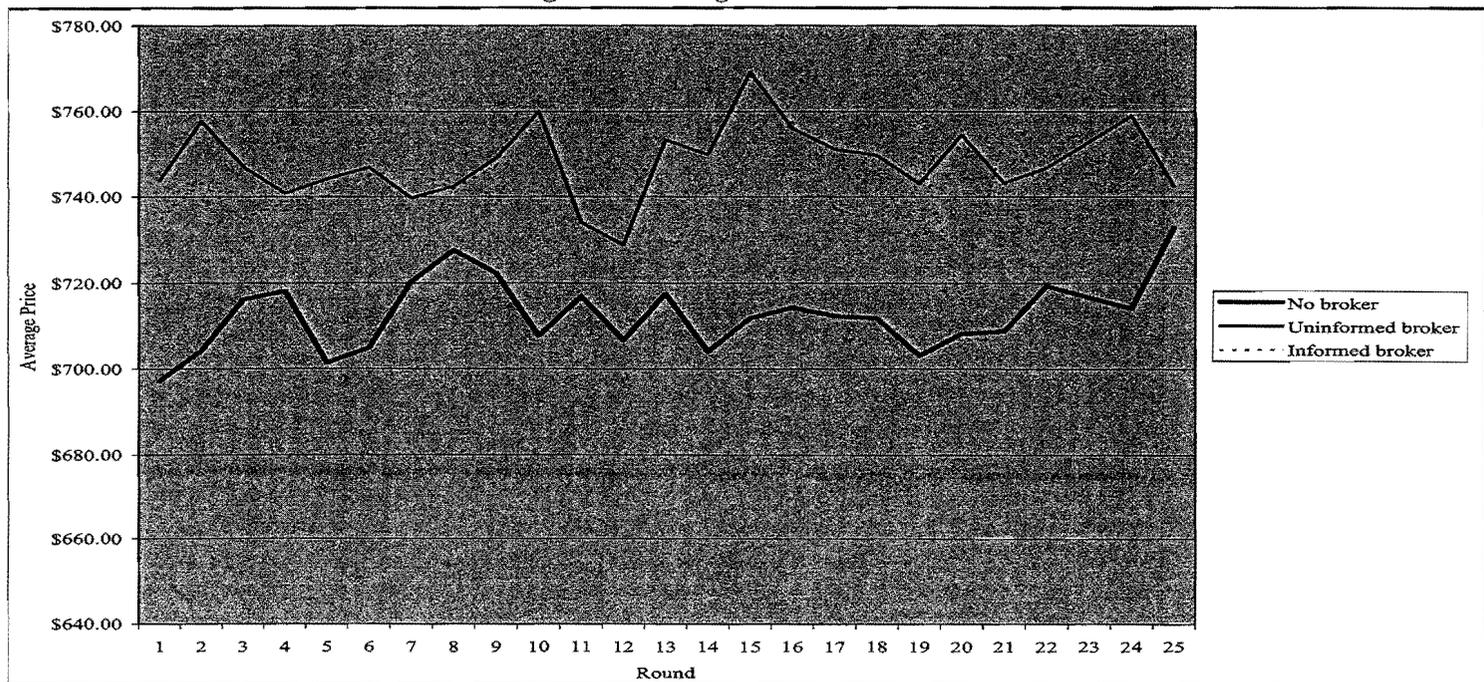
The significance of the role played by brokers is evident by the fact that, in most housing markets, they assist with the sale of more than 80% of single-family dwellings.

because the buyer knows that the broker represents the seller and gets a commission that is a percentage of the price.

The results of the broker's impact on the negotiated price are displayed in Figure 1. As the figure shows, the negotiated price was higher with a broker

the seller is willing to accept, while the buyer's initial bid price sends a signal to the seller about the highest price the buyer would be willing to pay. Thus, the buyer and seller are able to draw inferences about each others' reservation prices from the initial ask and bid prices.

Figure 1: Average Transaction Price



Real Estate Markets

We next looked at how the broker affects the success rate, and length, of the negotiation process. The broker, who is paid only if a sale occurs, has incentives to arrange a successful transaction as quickly as possible. One of the broker's potential roles is to "lubricate" bargaining during an impasse. We had expected this effect to be stronger with an informed broker, since a major source of disagreement in negotiations is that buyers underestimate sellers' reservation prices and/or sellers overestimate buyers'.

Players may try to reveal their reservation prices, but neither party has reason to trust the other's claims. As an outsider, the broker may lessen distrust between buyer and seller, thereby speeding up agreement. We had expected this effect to be stronger with an informed broker, but a mitigating factor is that, as noted, the broker represents the seller, and the commission is an increasing function of the price, so buyers may distrust brokers. But adding a broker reduces the number of rounds in the bargaining process, in terms of offers/counter-offers before agreement is reached. The implication is that brokers make each negotiating round more productive, evidence that supports the argument that brokers facilitate bargaining.

Finally, we studied how a broker affects the probability of a sale. Figure 2 compares the agreement rate for each period across the three sessions. The results

show generally that adding a broker, informed or uninformed, *reduces* the likelihood of an agreement compared to unmediated negotiation. This outcome is probably due to the smaller gains from trade between any buyer/seller pair because of the need to pay the commission.

Conclusion

Our experiments indicate that the participation of a broker increases the sale price and reduces the number of negotiating rounds needed to reach agreement. Offsetting this benefit is that brokers reduce the likelihood of a successful negotiation.

Clearly, our experiments focused on only one of several functions brokers fill in real estate transactions. In addition to facilitating sales, they help sellers and

with experiments. An example is the role of the buyer's broker. Experiments could be designed with a buyer directly represented by a broker; results could be compared to those reported here to capture the impact of buyer brokerage versus traditional seller brokerage. A related issue is whether buyer brokers should be compensated by buyers or sellers. Another extension might therefore be to design sessions to study the impact of alternative commission structures (flat fee, net listing systems) on negotiated prices. ■

Notes

1. Reservation price for the seller is the lowest price at which he or she is willing to sell the house, while the reservation price for the buyer is the highest price that he or she is willing to pay. These prices reflect how much the parties value the house, and what their alternatives are if no trade takes place. The seller's reservation price incorporates any brokerage commission that he or she must pay. This reservation price is not necessarily equal to the seller's listing price.

Experiments could be designed to compare buyer brokerage to traditional seller brokerage, and the related issue of whether a buyer's broker should be compensated by the buyer or the seller.

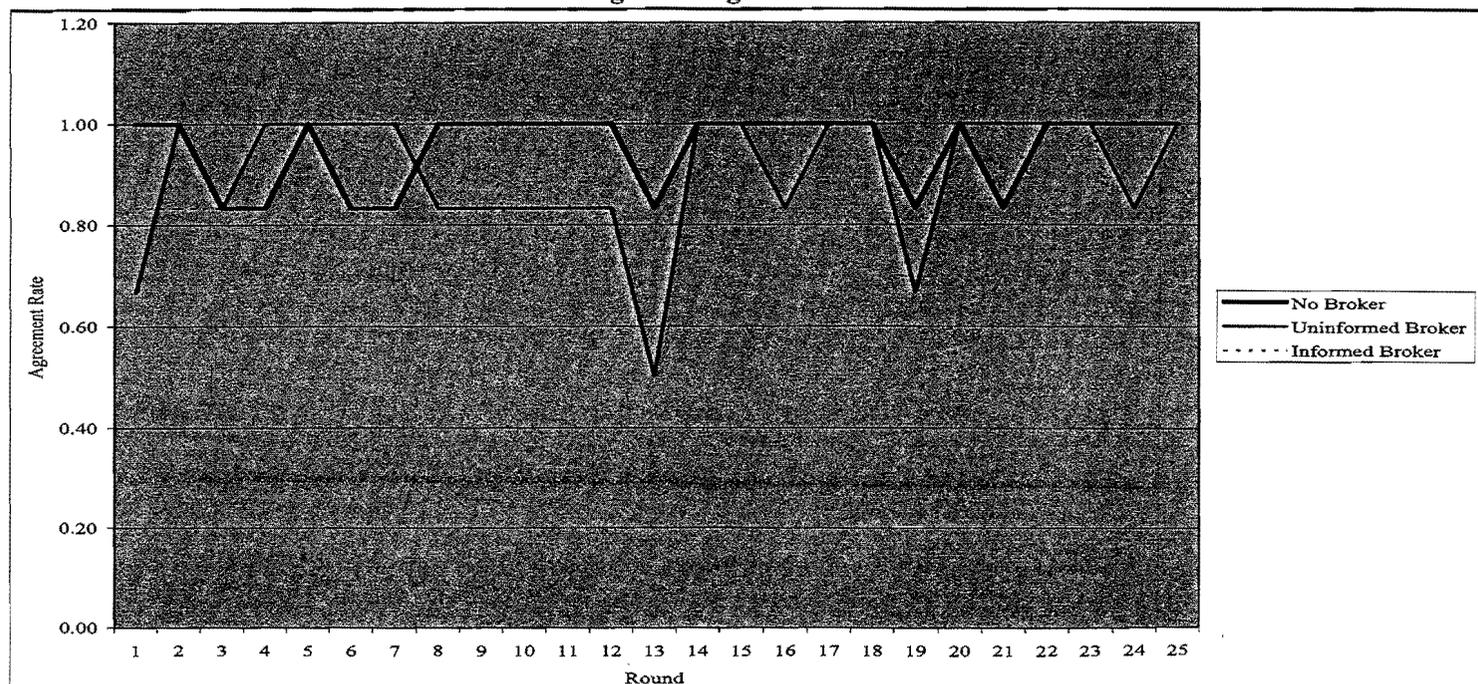
buyers find each other and assist with such issues as financing, inspections, and closing. The experimental methodology enabled us to isolate brokers' role in the negotiating stage from their other roles.

A number of other interesting real estate transaction issues could be tested

2. See Yavas, A., T. Miceli, and C.F. Sirmans, "An Experimental Analysis of the Impact of Intermediaries on the Outcome of Bargaining Games," *Real Estate Economics*, forthcoming.

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Figure 2: Agreement Rate



The Greatest Real Estate Movies of All Time: The Sequel

Austin J. Jaffe

My friend Steve Malpezzi performed a truly valuable service for others of us in the real estate field by writing "The Greatest Real Estate Movies of All Time" (*Illinois Real Estate Letter*, Winter 1999). His fine (and reader-acclaimed) review captured many special moments that real estate has accounted for in the history of the big screen. Yet his survey consisted almost entirely of American films (with only seven exceptions, by my count). Of course, the cinematic world, like real estate itself, reaches far beyond US borders. In this survey, I attempt to extend Steve's effort internationally, or "globally," as it is fashionable to say these days.

Missing Yankee Gems

Before leaving our domestic shores on this tour, we might want to identify a few American movies somehow omitted from Steve's list. One good example is *Of Mice and Men* [1939, 1992], with Steinbeck's California drifters dreaming of "living off the fat o' da land." In *This Land is Mine* [1943], an immigrant film maker from France gives Americans a view of an occupied European village: eminent domain in the extreme. *Home Alone* [1990] gives us more physical props and gags than any other movie since Buster Keaton's days, as the young hero protects his family's real estate. Real estate is also one of the stars of *Sunset Boulevard* [1950], though mention of the swimming pool in which viewers see William Holden floating face down might be discreetly omitted from a multiple listing sheet. By the end, we witness Gloria Swanson's "grand exit" down the large staircase, directed by her butler, who is played by Erich von Stroheim – a real-life film director.

While it might be hard to like *Blume in Love* [1973], there is a special moment at the movie's end. George Segal returns to Italy with the hope of seeing Susan Anspach at their special rendezvous spot: Piazza San Marco in Venice, one of the finest pieces of real estate on earth. I will cheerfully sit through two hours of boredom to see the magic of the wild pigeons, the uplifting violins, and the

historic cafés in front of the Basilica. In *Rear Window* [1954], Jimmy Stewart's view of the action across the courtyard must be the ultimate example of an externality affecting someone's enjoyment of their real estate. Or recall *The Fountainhead* [1949] and its cult-like architecture, which matches the movie's cult-like following in the years since it appeared. In *Boom Town* [1940], Clark Gable and Spencer Tracy remind us that natural resources contain valuable property rights worth fighting over. The more recent *Edward Scissorhands* [1990] reminds us, in its mystical way, that well-trimmed

Unimproved Land Around the World
What better way to begin our quest for great real estate moments in motion pictures worldwide than with Jean-Jacques Annaud's *Quest for Fire* [1982], which supports the academic tradition of property rights having originated in pre-historic times. The movie captures the problems of primitive mankind's search for shelter – a commodity that was just as important 80,000 years ago as it is in modern times – without benefit of the use of modern language. Difficulties of living on hostile land are depicted in Luis Bunuel's famous *Land Without Bread* [1932],

Home Alone [1990] gives us more physical props and gags than any other movie since Buster Keaton's days, as the young hero protects his family's real estate.

gardens are real estate, too. Even Jack Benny shows up in an old house in *George Washington Slept Here* [1942]. In a different mode, viewers can visit the *City of Joy* [1992] for a view of modern Calcutta, but this tour of some fairly dismal real estate is not for the squeamish. The list could go on and on.

Hollywood's latest contribution to those of us who love both the silver screen and real estate is *American Beauty* [2000]. Though not destined to be on everyone's list of favorites, the film offers a number of real estate connections. Carolyn (Annette Benning) is a real estate salesperson in a suburban world where it is very important to own a home. At one point in the story, there is a property dispute over the location of a tree. There are numerous disagreements between neighbors on the block, and several incidents of nosy neighbors using their real estate to watch each other through various windows. There is even competition for real estate closings. All these elements are integral to the story line. So despite – or perhaps because of – the rich history of real estate in American movies, this film is far from being passé.

Dovzhenko's *Earth* [1930], and other classic foreign films.

If *Earth* represents the Ukrainian, poetic vision for collectivization, Ken Loach's *Land and Freedom* [1996] provides the argumentative side of the debate, in this case, among the Spanish peasantry in the 1930s. Of course, issues associated with land reform have existed throughout history. Indeed, movies have long reflected the difficulties associated with poor, and often landless, families. In *Bicycle Thief* [1949], Vittorio de Sica provides a classic story of despair in post-World War II Italy. Everyday life is difficult in urban space in "neo-realist" Italy, even in the flea markets of Rome.

The unforgiving desert is another frequent setting for stories in the movies. The best examples are David Lean's epic for the big screen *Lawrence of Arabia* [1952]; Kiichi Ichikawa's enchanting allegory *Woman of the Dunes* [1964]; and Bernardo Bertolucci's *The Sheltering Sky* [1990], a disturbing tale set in North Africa. In each of these films, the desert – an especially interesting form of real estate, for reasons that include water rights issues – is the main character.

Similarly, life in the *jungle* is no “picnic” or “day at the beach” (after all, those types of real estate appear in other movies). For example, consider *Fitzcarraldo* [1982], Werner Herzog’s interesting story of a mad visionary’s dream to bring Grand Opera to the Amazon; or Frank Boorman’s *The Emerald Forest* [1985], a man-versus-nature parable; or Hector Babenco’s *At Play in the Fields of the Lord* [1991], another journey to Brazil with its attendant risks and ultimate disasters. Another setting of hostile topography is in the *mountains*. International cinema’s best example is the relatively unknown Sergei Bodrov film *Prisoner of the Mountains* [1996]. Not only does the land seem isolated and remote; so do the characters. Unusual real

Traveling to acquire land, usually in America, is, in fact, a repeated theme for foreign movie makers. Consider Jan Troell’s classic *The Emigrants* [1970], in which Swedes head to Minnesota; or Gregory Nava’s influential *El Norte* [1983], in which South Americans travel north to find riches. In Ron Howard’s disappointing (to critics, not to me) *Far and Away* [1992], poor emigrants (a pre-divorce Tom Cruise and Nicole Kidman) leave Ireland and eventually stake homestead claims in Oklahoma. Of course, land grabs and their attendant benefits are a strong theme in so many real estate movies; who could forget Herzog’s *Aguirre, Wrath of God* [1972], with Klaus Kinski going AWOL from Pizarro’s army in 1560 and heading up river in

country-like settings can be found in de Sica’s *The Garden of the Finzi-Continis* [1970] with the unwanted rise of Fascism; or in Fernando Trueba’s modern Spanish classic *Belle Époque (The Age of Beauty)* [1992], in which an old land owner’s four available daughters show much interest in the charms of a young Spanish Civil War deserter. Or consider the controversial final work of Stanley Kubrick, in which the British film maker worked with Cruise and Kidman, this time with their *Eyes Wide Shut* [2000]. Despite all the advance press about the bedroom scenes, the best part of the film is the real estate; the venue is a mysterious house located in the countryside just outside New York City.

Film makers love *islands* as places to tell their stories. Of course, an island, perhaps even more than other real estate, can easily become the key ingredient in a film. Some international favorites are Michelangelo Antonioni’s *L’Avventura* [1960], a true modern classic that begins on an island. Or recall the plane loads of children unexpectedly abandoned on a tropical island in Peter Brook’s *Lord of the Flies* (the 1963 UK original is much superior to the 1990 US remake). There is also Lina Wertmüller’s *Swept Away* [1975], where Italian politics and class distinctions give way to survival skills and sexual feelings on an idyllic island.

More recently, *Mediterraneo* [1991], directed by Gabriele Salvatore, is another Italian island adventure. Here a lost company of soldiers waits out the war somewhere in Greece; there are worse places to put in army time, and some of the troops can never leave their newfound real estate. In Michael Radford’s *The Postman* [1995], a dim-witted but likable fellow learns to write poetry to win the heart of a local beauty, as the world constrains them to the island where they were born. Foreign film makers have the ability to show the warmth and tranquility of island life, even if the island has been spoiled by “civilization,” and even if we see the film in the dark of winter. Indeed, being “swept away” by foreign cultures, and to exotic, faraway places, is what movies are all about. In real estate terms, watching foreign movies is like taking an inexpensive property tour, but without the high-pressure sales pitches.

Quest for Fire [1982] captures the problems of primitive man's search for shelter – a commodity just as important 80,000 years ago as in modern times.

estate characteristics will do that to people. (I can not resist plugging a favorite American film, not mentioned before, that is sited largely in the mountains: Frank Capra’s *Lost Horizon* [1937]. Take me to Shangri-La any time.)

In Stefan Ruzowitzky’s *The Inheritors* [1998], a small group of 19th century Austrian peasants become “farmers” (*i.e.*, land owners) unexpectedly when the farm they worked is left to them. Real estate issues abound: in a group of one-tenth (then one-seventh) equal owners, who is in charge? How will the land be worked? Suddenly, we see pride of ownership with the change in tenure status on the property. Title to land matters a great deal, whereas previously it had never been a consideration. At the same time, jealous neighbors can not come to grips with the new property owners (“once a peasant, always a peasant”). The “tragedy of the commons” is a strong possibility in the beginning, but it is fought off by hard work, only to lead to other tragedies. *The Inheritors* is much like *Animal Farm* [1965], gone even wilder than in Orwell’s vision. In the end, the survivors follow their dream to secure land in America.

search of land to stake a claim for gold? You might recall the scores of monkeys on the raft more than anything else.

More pleasant experiences are found in the frequent cinematic dreams of living in the *countryside*. For example, Jean Renoir’s lovely *A Day in the Country* [1936] is virtually a tribute to his father, Impressionist painter Auguste. Bertrand Tavernier’s *Sunday in the Country* [1984] is similar, but was made a generation later. Another Renoir masterpiece, *The Rules of the Game* [1939], is largely based around a ritualistic fox hunt (with a famous anti-hunting montage) at an elaborate hunting camp in rural France. In one of Ingmar Bergman’s famous masterpieces, *Wild Strawberries* [1957], there is a touching scene when old Isak finds himself alone in a strawberry patch. It is peaceful and tranquil, but Isak is sad, empty, and confused. (This is also perhaps the best film to see to help you gain an appreciation of the famous “Swedish daylight,” the unique, flat sunlight found only in the Nordic countries during the long summer months.)

Of course, rural tranquility is not always the order of the day. Trouble in

Special-Purpose Improvements

Movies have long made use of assorted property types in illustrating the well-known point that when land is developed, it can be transformed to fit the special needs of various users. For example, *apartments* are the central theme in Roman Polanski's *Repulsion* [1965], and in Bertolucci's *Last Tango in Paris* [1972], but there are many others, as well.

Traditional French directors seem to seek out stories in *hotels*: see Marcel Carné's *Hotel du Nord* [1938], or perhaps Renoir's *La Bête Humaine* [1938]; there is nothing like watching Jean Gabin hanging around a sleazy French hotel between the wars. Another hotel movie is Alain Resnais's *Last Year at Marienbad* [1961], though the real estate in question is more resort than hotel, in light of the numerous shots of painted ceilings and tapestry-covered walls. (I must confess, however, that my favorite part of *Marienbad* is when the mysterious guest challenges others to play a game that he can not lose. I taught my daughter this game, and now she never loses, either.) Indeed, many of the so-called French New Wave directors have found gems in small Parisian hotels.

Another international cinematic tradition is to use *schools* as a focal point. A personal favorite is Josef von Sternberg's *The Blue Angel* [1930]. My favorite ending (from among at least three possible alternatives) has former Professor Rath (Emil Jannings) returning to his beloved classroom to expire from the shame of his infatuation with ordinary cabaret singer Lola Lola (Marlene Dietrich). But the ultimate French classroom movie is *The 400 Blows* [1959], Francois Truffaut's partial remake of French film legend Jean Vigo's forgotten classic *Zero for Conduct* [1933]. These classroom films have been models for many others, including portions of Federico Fellini's films via flashbacks in post-World War II Italy. Two British entrees to the genre are Tony Richardson's *The Loneliness of the Long Distance Runner* [1962] and Lindsay Anderson's *If ...* [1968]; rebellion runs strongly through both movies (as it runs through Sidney Poitier's tough young charges in an English high school in *To Sir, With Love* [1966]). More recently, controversial director Marco Bellocchio's

Devil in the Flesh [1987] is centered on a teenage student's involvement with an "older" woman; he often jumps right out of his classroom window to meet her.

Because European culture is fond of small *shops*, we should note Jacques Demy's *The Umbrellas of Cherbourg* [1964] for the local garage and, later, a new service station, as well as the town's best place to buy umbrellas. Another good example is Jan Kadar's *The Shop on Main Street* [1965], the Czechoslovakian original version of the quaint story of two awkward souls trying to get together. Remakes include *The Shop around the Corner* [1940] (directed by a Hungarian immigrant to the US, Ernst Lubitsch) and Nora Ephron's *You've Got Mail* [1998], the modern e-mail based US version.

Because music plays such a key role in foreign films (entire books are devoted to the choice of music in the movies), it is not shocking that *opera houses* have been the settings for many international films. In addition to *Fitzcarraldo*, mentioned earlier, there is Bergman's beautiful version of Mozart's *Magic Flute* [1974]; Jean-Jacques Beineix's *Diva* [1981] (which begins on the opera stage, as do many other films); and Gérard Corbiau's *Farinelli: Il Castrato* [1994], an interesting story, though an unusual one, to say the least. (I also love *Amadeus* [1984], another American film. Perhaps because much of it was shot in Prague and director Milos Forman is a Czech immigrant to the US, it seems appropriate to note the work here.) For a different take on opera, see Kaige Chen's *Farewell my Concubine* [1993], part of China's New Wave of films and a wide-ranging story involving Chinese opera over several years.

Without films, what would younger generations know about *dance halls*, important real estate from a bygone era? In one of Alfred Hitchcock's classics, *The 39 Steps* [1935], the finale unwinds on stage. Or marvel at Carné's masterful tribute to the theater, *Children of Paradise* [1945]. A pre-immigration Forman directed *The Fireman's Ball* [1967]; Ettore Scola's *Le Bal* [1982] has no talking, only singing and dancing. Masayuki Suo's delightful *Shall We Dance?* [1996] shows modern Japan's fascination with ballroom dancing, though the championship competition is held in England.

Moving to the seedier side, *brothels* of some sort are a commonly featured venue in international cinema. Kenji Mizoguchi's *Street of Shame* [1956] is an example of Japan's look at urban prostitution, but it is far from the only one. Bunuel's *Belle de Jour* [1967] is perhaps famous because of Catherine Deneuve's dual role in what has been termed a "fascinating fantasy" for many viewers. In *Amarcord* [1973], Fellini "remembers" the milieu of prostitutes in post-World War II Italy with adolescent fondness and a child's curiosity; this attraction spills over into many of his other films, as well. Indeed, many film makers, especially French directors, seem fascinated with brothels, their employees, and their customers. (Any reference here to an American movie featuring a Texas "ranch" in the title, or Burt Reynolds and Dolly Parton in starring roles, would be in terribly poor taste.)

Many other special forms of real estate are featured in international films. These include a *restaurant* in Louis Malle's acclaimed *My Dinner with André* [1982], a women's *bathhouse* in Joseph Losey's *Steaming* [1985], and a *riverboat* in Vigo's classic "home on the barge" *L'Atalante* [1934]. We see more than we might wish of Polish *sewers* in Andrzej Wajda's *Kanal* [1956], while other films, including every version of *Les Misérables*, take us to the sewer systems of cities around the world. We spend time in an Israeli *commune* while viewing Eli Cohen's *Under the Domin Tree* [1994]; while it is in a *courthouse*, as we might have expected, that Kafka's hero is trapped by the entire legal system in Orson Welles's *The Trial* [1962]. *Museums* often share screen time with the actors, as well; for example, see Bellocchio's *The Conviction* [1994] or Léos Carax's *The Lovers on the Bridge* [1999].

A final special purpose property featured in moviedom is the beloved *cinema* itself. Directors are said to be in love with the movies, and viewers are invited to become voyeurs. Many films pay tribute to the art of movie making, but none are as devoted to the movie house as a central figure as is Giuseppe Tornatore's *Cinema Paradiso* [1988]. (1971's *The Last Picture Show* offers a slightly more tasteful reference to Texas and American films.)

Trapped in Urban Life, or Rural

Because the study of real estate springs from urban economics, I think of movies about characters struggling with life while trapped within cities as being real estate-related. In the famous *The Cabinet of Dr. Caligari* [1920], Robert Weine and the German Expressionist movement come of age by demonstrating the style and artistic eloquence of shapes, sizes, shadows, and lights in urban space. City landscapes were never quite the same ever since *Caligari*. Interestingly, in recent visits to Eastern and Central Europe I have seen "transition" cities exude the same expressionistic feelings. For example, walking around in Riga, Vilnius, and some other cities at night reminded me of this period in film art.

No wonder the grass, and the land, can look greener on the country side of the fence. Fellini's *Amarcord* [1973] is about remembering life in a small town in Italy; it seems that each of his later movies is about his life experience in some small Italian town. Strangely, the viewer gets the impression that Fellini was more than pleased to get *out* of these towns in real life. Indeed, small towns are not a sure solution to the real estate world's "location, location, location" dilemma. In Bertolucci's *The Last Emperor* [1987], the child ruler is trapped in the Forbidden City, essentially a small town within populous Beijing. Of course, this is not a parable, but true history from the 1930s and 1940s. In Michael Verhoeven's controversial *The Nasty Girl*

Akira Kurosawa's hugely popular *Seven Samurai* [1954], with 16th century Japanese villagers hiring samurai warriors to protect their property from annual bandit raids; and *Stalingrad* [1993], with Joseph Vilsmaier detailing the grim legendary World War II urban battle between German and Russian forces, in which more than two million lives were lost.

Other international films about battles over small or large plots of land are Sergei Bondarchuk's *War and Peace* [1954], the expensive USSR version of Tolstoy's novel; Peter Weir's *Gallipoli* [1981], about Australia's participation in World War I and the politics of holding ground in combat; and Herzog's *Lesson in Darkness* [1992], a documentary about Kuwaiti oil fires from the Gulf War. None of these are happy stories, but disputes over territory can become quite bitter, and film makers do not shy from showing us the bullets, guts, and blood.

There is a special tradition of movies depicting civil wars, perhaps the ultimate variety of dispute over domestic land. A famous "newsreel" is Eisenstein's *October* [1927], which captures the 1917 Bolshevik revolution. The Napoleonic wars are the backdrop for Stendhal's classic novel in the disappointing film, Claude Autant-Lara's *The Red and the Black* [1954] (classic literature is said to generally be infinitely better than the film it inspires). Gillo Pontecorvo's *The Battle of Algiers* [1966] is another example of civil war's effect on city life. Loach's *Land and Freedom* [1996, discussed earlier] deals with the Spanish Civil War and its impact on fighters in the rural countryside. Chen's *The Emperor and the Assassin* [1999] illustrates the horrors of everyday life at the Chinese Empire's founding in 200 BC. Try to forget the horrors when watching any of these civil war movies; marvel at the staging of the battle scenes.

With wars come stories of prisoner-of-war camps and concentration camps. My favorite POW movie is Renoir's *Grand Illusion* [1937]. This very stylized and complex work anticipates the coming war, but is set in World War I. Many of the film's unforgettable moments involve real estate matters. Another example is Caspar Wrede's *One Day in the Life of Ivan Denisovich* [1971], Solzhenitsyn's account of prison camps, Stalinist-style.

Issues associated with land reform have existed throughout history. Indeed, movies have long reflected the difficulties associated with poor, and often landless, families.

In *M* [1931], Fritz Lang's masterful depiction of a child murderer (Peter Lorre before his Hollywood fame), urban space is portrayed as a prison. A famous scene in the beginning has a child's balloon getting trapped in the phone wires. By the end, Lorre is trapped by Berlin's underworld of gangsters and thieves, rather than finding justice with the police. Not only is he trapped by his twisted mind; it is also urban life that Lorre can not escape. Trying to escape from urban *pov-erty* is a frequent theme in several foreign films. Two classic examples are Luchino Visconti's *La Terra Trema* [1948], a film about everyday life and its difficulties in Sicily; and Satyajit Ray's *World of Apu* [1958], about urban life in India.

Vasili Pichul's *Little Vera* [1989] shows urban life as pretty grim in post-Soviet Russia, with nowhere for people to go. Similarly, Eugeny Mihaylov's harsh *Canary Season* [1998] depicts the "lost generation" in post-Communist Bulgaria. Transition in Eastern Europe has not proven as easy as some pundits had hoped. But cities can be humorous, as well. Jacques Tati's famous *Playtime* [1967] looks at the insanity of city life.

[1990], a young woman is trapped by attitudes of the older people in a small German town when she discovers secrets about their wartime pasts.

War as a Property Dispute

Almost every war can be viewed as a dispute over property rights. Fighting for control of real estate and other valuable property is a time-honored tradition, dating back to antiquity. A dictator's laying claim to land beyond his national boundary is not unlike a private party's laying claim to land beyond his own property's border. Thus, disputes over boundaries figure prominently in the history of cinema; the battles have often been fully staged and gloriously filmed.

A classic example of an international film with a battle to control land as its major objective is Sergei Eisenstein's *Alexsandr Nevsky* [1938], with its famous Sergei Prokofiev score, in which the Battle on the Ice (it took place April 5, 1242, if you wondered) pitted Russian troops against a German aggressor. Other films devoted to battles over land include Roberto Rossellini's *Open City* [1945], about World War II's final days in Rome;

Many concentration camp films have been produced, not all of them in recent years. Consider two, at opposite ends of the spectrum. Fellow film maker Truffaut has called Resnais's famous documentary, *Night and Fog* [1955], the greatest movie ever made. On the other hand, Roberto Benigni's *Life is Beautiful* [1998] is a comedy (of sorts) about surviving the concentration camp experience. The film is much better than it sounds.

Praising Urban Life, Futuristic Cities
Critics suggest that many foreign films are best viewed as testimonials to cities where the characters (and directors) find themselves living. Such a movie might be like a love letter to a city itself, or like a distant documentary of life in a specific city as discovered by the film maker. As examples, consider *Breathless* [1959], Jean-Luc Godard's tribute to American gangster movies that is perhaps better viewed as a salute to the city of Paris. Of course, many Godard and Truffaut movies are mostly about Paris (like Fellini's are largely about Rome and Mizoguchi's are often about Tokyo).

There is also the enchanting *Black Orpheus* [1958], directed by Marcel Camus, about the Carnival in the slums of Rio de Janeiro; or Visconti's *Death in Venice* [1971], whose opening scene with Dirk Bogarde riding into the city on a gondola (allegedly with authentic costumes) sets the stage for the decadent story to come. Fellini's *Roma* [1972] is a love letter to the Eternal City; in the best scene, ancient wall paintings evaporate when confronted by open air due to evacuations. *Alexanderplatz* [1980] is Wim Wenders's massive monument to Berlin, while *Lisbon Story* [1995] is his fond depiction of Portugal's capital. There are many such paeans to urban life.

Science fiction movies were invented outside the US. A famous early example is Lang's classic *Metropolis* [1927]; the film has been enormously influential, as noted in Malpezzi's article. An interesting footnote is that Lang marked the "future" as the year 2000. The realistic side of film was also founded in France, by the Lumière Brothers who, among other things, produced *Workers Leaving the Lumière Factory* [1895] as their first film, and one related to real estate.

The Final Frontier

Space travel is a quest for new land if ever there has been one. Thus it is interesting that one of the founding fathers of the artistic side of French cinema was Georges Méliès. His *A Trip to the Moon* [1902] is not just another pioneering science fiction film; it is the spiritual predecessor for all other space travel movies.

Two other works that focus on truly distant real estate might be mentioned. Kubrick's *2001—A Space Odyssey* [1968] made a big splash internationally. Less famous is Andrei Tarkovsky's epic *Solaris* [1971], about a space station and communication with new worlds. "Sci-fi" has not been the same since, and even real estate has changed as the specter of HAL inhabits our ever-smarter buildings.

Traveling to acquire land in America is a repeated theme for foreign movie makers. Consider The Emigrants [1970], in which Swedes head to Minnesota; or El Norte [1983], in which South Americans travel north to find riches.

The Top Ten (OK, Fourteen) Real Estate Moments in Foreign Movies

As a grand finale, I propose to identify, and then briefly describe, my "top ten" (actually, top fourteen; all are so good that I could not choose four to cut out) real estate moments in international cinematic history. Readers are free to disagree, but I find all of these scenes (some of which occur in films that are mentioned above) to be very special, and beg your indulgence, having sat in the dark for more than 25 years putting this list together. So, in no particular order, here are my top real estate moments in the world of international film.

Battleship Potemkin [1925]. Eisenstein's classic war story is generally remembered for, among other things, the baby carriage falling down the Odessa steps. This scene has been called possibly the most famous movie scene of all time. It seems ironic that one of film's ultimate real estate moments should occur in Soviet Ukraine. Of course, *The Untouchables* [1987] pays tribute to this scene using a flight of stairs in a more capitalistic parcel of famous real estate: Chicago's Union Station.

The Seventh Seal [1957]. One of Bergman's classics tells of the search for the meaning of life in the Middle Ages (of course, which Bergman film is *not* about the search for life's meaning?). In one especially famous scene, the Knight (Max von Sydow) plays chess with Death. The famous real estate scene that gets this film on the list is when the entire dance troupe is silhouetted against the sunset, following Death up the hill at the end of the picture.

The Beauty and the Beast [1946]. Long before Walt Disney's animated version, there was the classic French version, directed by Jean Cocteau and starring Jean Marais as the Beast. In this stunning film, the castle is highlighted by faces in the

walls, and arms holding the candles as the actors move from room to room. No wonder Belle becomes enchanted.

Raise the Red Lantern [1991]. In one of the widely heralded New Wave Chinese movies, directed by Zhang Yimou, a wealthy property owner in 1920 provides four separate houses on his large estate, one for each of his four wives. Every evening a red lantern is posted to signal where the master, Mr. Chen, intends to spend the night. Songlian (played by Gong Li) has to compete in a power struggle for his affection; it is hard to imagine that she could lose.

Walkabout [1970]. This is Nicholas Roeg's tale from the lush and wild Australian outback. My favorite moment is when the English girl takes a step back onto the paved road. The camera shoots the girl as she stops before the road. It pans down to her feet as she hesitates a moment before leaving the wild, and then shows her symbolically returning to the improved real estate that represents modern life. It is truly a classic moment in film history!

Just For Fun

Ikiru (To Live) [1952]. Kurosawa tells the story of a minor civil servant who learns that he has a terminal illness, and decides to work to get a local park created through the bureaucracy. The shot of him at peace with his life, swinging in the park in a snowstorm to celebrate his small but important lifetime achievement, is unforgettable.

Fitzcarraldo [1982]. Herzog provides this haunting film about a dreamer and his quest to bring opera to the Amazon jungle. The best moment: faced with hostility as they travel down river, Fitz plays Caruso for the warring natives on his gramophone in an attempt to drown out the war drums, and to settle the fears of his men.

Weekend [1967]. This film is one of Godard's frequent indictments of "modern" capitalism (imagine what he might do with the dot.com age). The best scene is the long traffic jam (people traveling to get to and from their real estate), shown as a metaphor for what has gone wrong with contemporary life.

Jules and Jim [1961]. In this irresistible Truffaut film, Catherine (played by Jeanne Moreau) is torn between her two best friends (Oskar Werner and Henri Serre). There are several memorable scenes, but for me, the instant when Catherine jumps into the Seine River on a whim not only captures the spirit of the film, but raises interesting questions on riparian rights, as well.

Virgin Spring [1959]. Yet another classic from the Swedish master Bergman makes my list. The highlight occurs when the child is found; a pure spring bubbles up from the spot where the murder has occurred. Land is that special, sometimes.

The Third Man [1949]. In Carol Reed's classic story about post-war struggles in Vienna, Harry Lime (infamously played by Orson Welles) is eventually found by Joseph Cotten, an old friend. They agree to meet atop an old Ferris wheel (still operating today), since Cotten can not believe that Welles has become a notorious drug dealer. It is reported that Welles was plagued for years by fans who

thought he truly believed the movie speech he made on the Ferris wheel:

"Look down there. Would you really feel any pity if one of those dots stopped moving forever? If I offered you twenty thousand pounds for every dot that stopped, would you really, old man, tell me to keep my money, or would you calculate how many dots you could afford to spare? Free of income tax, old man, free of income tax. It's the only way to save money nowadays."

Viridiana [1961]. The film offers viewers Bunuel's indictment of class manners and customs (all of his films do this, more or less). The most famous scene here is the crude parody of the Last Supper. The original da Vinci painting in a Milanese church has always been difficult to view

Raufenstein fatally shoots Boeldieu, the commandant is visibly distraught, not with sadness over having to abide by his devotion to military duty, but by the loss of a fellow landed aristocrat and kindred spirit from an era gone by. Raufenstein returns to his dank, cold room where only one semblance of life remains: a potted plant. With great finesse, he takes a scissors and cuts the bloom off the flower. Life has been snuffed out now for all.

Conclusion

An old axiom is that movie sequels are never better than the original productions (a view generally agreed on at least until *Godfather II*). Perhaps it holds for these two "real estate and the movies" pieces, as well. No problem; I will not mind if

Despite all the advance press, the best part of Eyes Wide Shut [2000] is the real estate; the venue is a mysterious house in the countryside just outside New York City.

in person, despite several restorations. One of film's great powers is that it can actually re-create real estate.

The Lovers on the Bridge [1999]. This recent movie by Carax has achieved notoriety because it is, apparently, the costliest French film ever made (they had to reconstruct a bridge, the famous Pont Neuf over the Seine in Paris, due to scheduling problems). In this strange love story, Michele and Alex, two homeless souls, dance across the bridge in a fabulous fireworks display for several minutes, but it seems like forever.

Grand Illusion [1937]. My personal favorite is this truly classic French war movie by Renoir. The film features an abandoned old castle, which Prussian officer von Raufenstein (played by von Stroheim) is assigned to occupy, and to use occasionally as a prison. There are many classic real estate moments. The best occurs after the commandant's new orders are to guard three French prisoners, one a fellow professional officer (Boeldieu, played by Pierre Fresnay). After a chase during an escape, where

readers conclude that I came in second to Steve Malpezzi, a solid real estate scholar and avid, long-time movie fan.

The only remaining issue is when, and where, we might contract for a dark facility at a key location in urban space, anywhere in the world, where as many as hundreds of our neighbors will impose a number of personal external effects on us, to our delight and chagrin. Of course, we will not be making a real estate decision; we'll all be going to the movies! I hope to see you there. ■

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Teaching Annuities With One Simple Equation

Joseph W. Trefzger

Financial calculators and spreadsheets are wonderful tools, but are of limited value until the user can structure solutions with understanding of the underlying concepts. Too many students think they know time value when they can hit key sequences while following textbook instructions. Such individuals have great difficulty finding solutions, or even identifying problem types, when situations are not explicitly identified. After seeing confusion among even the best real estate students over the years, I developed a simple yet comprehensive way to classify and present essential time value ideas to those who must learn to analyze complex cash flows. Students need only the ability to handle basic exponents and logarithms.¹

One area of difficulty is identifying situations toward structuring equations that produce correct solutions. Annuities, with their many applications and range of possible unknowns to solve for, can seem especially incomprehensible. Indeed, the typical introductory finance text coverage, presenting many “different” situations that must be handled in different ways (along with authors’ typical insistence on identifying problem types so students see no need to learn to identify them on their own), can lead to frustration or resignation. Yet it need not be this way; sufficient similarities exist across annuity situations that we can develop a pedagogical tool much simpler than the ones textbooks continue to present.

A Very Versatile Equation

Students have little trouble distinguishing an annuity situation from a case in which two single dollar figures are compared. They do have difficulty distinguishing future value of annuity (FVA) from present value of annuity (PVA) situations, but there is a straightforward way to tell them apart, and the approach presented below facilitates this distinction. Each annuity example has a series of equal (or steadily changing) dollar payments or receipts (*cash flows*, or CFs), a rate of return r , a number of time periods n , and – the key – a *lump sum* of money that is equivalent, in time value-adjusted terms, to the cash

flow series. We identify the situation as FVA or PVA by looking at *when the lump sum* (be it a known or unknown amount) *exists intact*. If the known or unknown lump sum is intact today, in the *present* (as with a loan to repay), we have a PVA situation. If that lump sum will not exist intact until a *future* date (as in saving for retirement), we have a FVA situation.²

Still, annuities are harder to understand than single dollar situations. One complication is that the PVA factor is not merely the FVA factor’s reciprocal;³ thus, our approach must be sufficiently general that we can insert the correct factor after identifying when the lump sum that corresponds to the series of CFs is intact. Another complication is that annuity CFs can change hands at the end (*ordinary annuity*) or beginning (*annuity due*) of each period; we must adjust the standard (ordinary) factor if beginning CFs are expected. In other words, we can encounter ordinary annuity and annuity due situations in both FVA and PVA cases, and our approach must be flexible enough to deal with any possibility.⁴ A final complication is that a present or future lump sum may correspond to a series of CFs that are not equal, but change in some way over time (perhaps IRA deposits or a “step-up” lease with CFs that rise with inflation).

Our approach is a simple, but very versatile, equation – one that can be used in all FVA or PVA situations, ordinary or due, with level CFs or those that change by constant percentages. The equation is

$$\text{Payment} \times \text{Factor} = \text{Total}, \text{ or}$$

$$\text{PMT} \times \text{FAC} = \text{TOT}.$$

The regular CF (a PMT made or received each year or other period) times an annuity factor FAC equals the lump sum TOT to which the CF stream is equal, in time value-adjusted terms. FAC is the FVA or PVA factor, for an ordinary annuity or annuity due, involving level or changing CFs, as called for by when the lump sum is intact and when the CFs change hands.

Simpler Applications: Level Annuities

The ordinary annuity factors for level annuity situations (all CFs equal) are

$$\frac{(1+r)^n - 1}{r} \quad (\text{FVA})$$

$$\text{and} \quad 1 - \frac{\left(\frac{1}{1+r}\right)^n}{r} \quad (\text{PVA});$$

the latter simplifies to $1/r$ in the level *perpetuity* case involving the PV of a CF stream expected forever. An annuity due factor is the ordinary level annuity factor multiplied by $(1+r)$, since beginning CFs would cause compounding or discounting once more during the annuity’s life:

$$\frac{(1+r)^n - 1}{r} (1+r) \quad (\text{FVA})$$

$$\text{and} \quad \frac{1 - \left(\frac{1}{1+r}\right)^n}{r} (1+r) \quad (\text{PVA});$$

or $(1/r)(1+r)$ for PV of a level perpetuity with beginning CFs. $\text{PMT} \times \text{FAC} = \text{TOT}$ helps students understand concepts and handle computations involving all these types of level annuities, all routinely encountered in real estate applications.

Future Values of Annuities

Example 1: Lump Sum Value is Unknown

How much will be in your GNMA mutual fund by the end of year 5 if you put \$100 in each year and earn a 7% annual return on the growing balance? TOT will not exist until the end of year 5, so we have a FVA problem. With year-end deposits:

$$\text{PMT} \times \text{FAC} = \text{TOT}$$

$$\$100 \times \frac{(1.07)^5 - 1}{.07} = \text{TOT};$$

TOT = \$575.07. With *beginning* deposits:

$$\$100 \times \frac{(1.07)^5 - 1}{.07} (1.07) = \text{TOT};$$

TOT = \$615.33, higher than if we waited a year before our first deposit. Here the annuity due answer is simply the ordinary annuity answer times $(1+r)$, but that outcome is correct only if the unknown is the lump sum. Textbooks frustrate students with a “multiply vs. divide” quandary, but there need be no confusion. Just solve the $\text{PMT} \times \text{FAC} = \text{TOT}$ equation, and any unknown’s solution falls into place.

Instructor Notes

Example 2: Cash Flow is Unknown

You hope to have \$575.07 at the end of year 5. If the growing balance earns 7% annually, how much should you put in each year? For level year-end deposits:

$$PMT \times \frac{(1.07)^5 - 1}{.07} = \$575.07 ;$$

PMT = \$100. For *beginning* deposits:

$$PMT \times \frac{(1.07)^5 - 1}{.07} (1.07) = \$575.07 ;$$

the annuity due PMT = \$93.46 is *smaller* (we in effect *divide* the ordinary level annuity answer by 1.07). If we make the first deposit right away, more interest accrues over the account's life, so we can put less in each time and still reach the \$575.07 goal. PMT x FAC = TOT thus eliminates confusion over computations in end vs. beginning of period situations.

Example 3: Rate of Return is Unknown

If both PMT and TOT are known, we will be solving for one of the two possible unknowns (r , n) within FAC. You deposit \$100 at the *end* of each year; if you have \$575.07 by the end of year 5, what annual return did the mutual fund provide?

$$\$100 \times \frac{(1+r)^5 - 1}{r} = \$575.07 ;$$

the solution is $r = 7\%$. Finding r requires *trial and error* calculations (due to complications of having both r and r^5 in the same equation), but PMT x FAC = TOT facilitates the process, helping students see what they are solving for (and enter values into spreadsheets or financial calculators, which can do trial and error instantaneously). With *beginning* deposits:

$$\$100 \times \frac{(1+r)^5 - 1}{r} (1+r) = \$575.07 .$$

The solution is $r = 4.7\%$; amassing only \$575.07 if we give up money at the start of each period represents a lower return.

Example 4: Time Periods are Unknown

You deposit \$100 at the end of each year, and expect a 7% annual return on your growing balance. How long should it take for the account to reach \$575.07?

$$\$100 \times \frac{(1.07)^n - 1}{.07} = \$575.07 ;$$

$n = 5$. It takes 5 years for \$100 year-end deposits to grow to \$575.07 if we earn 7% annually on the growing balance. We

could find n with trial and error, or solve directly using logarithms.⁵ Either way, solving is a matter of simple algebra once the known values are correctly entered in $PMT \times FAC = TOT$. For the annuity due:

$$\$100 \times \frac{(1.07)^n - 1}{.07} (1.07) = \$575.07 ;$$

the solution is $n = 4.71981$ periods. Fewer years are needed to reach a given total if we make the first deposit immediately.

Present Values of Annuities

Example 1: Lump Sum Value is Unknown

You can afford to pay \$100 per year for five years; if you pay 7% annual interest, how much can you borrow?⁶ Lump sum TOT exists intact today, so we have a PVA problem. For year-end payments:

$$\$100 \times \frac{1 - \left(\frac{1}{1.07}\right)^5}{.07} = TOT ;$$

TOT = \$410.02. For payments made at the *beginning* of each period:

$$\$100 \times \frac{1 - \left(\frac{1}{1.07}\right)^5}{.07} (1.07) = TOT .$$

The answer (TOT = \$438.72) for beginning CFs is *greater*; we could service a larger loan in the unusual case of making the first payment immediately rather than at the end of the first period. As in the FVA case, $(1+r)$ times the ordinary level annuity answer yields the correct annuity due answer if the lump sum is unknown, but obviously not if the CF is unknown. But PMT x FAC = TOT lets us easily find the correct solution for any unknown.

In the perpetuity case, FAC is simply $1/r$ (PV is just the expected periodic CF divided by the required periodic return):⁷

$$\$100 \times \frac{1}{.07} = \frac{\$100}{.07} = TOT ;$$

paying \$100 at the *end* of each year *forever* on a 7% loan, you could borrow TOT = \$1,428.57. A perpetuity's principal is left intact, with each period's CF consisting only of the rate of return. If CFs begin immediately, the factor $1/r$ is, true to our pattern, simply multiplied by $1+r$ (the equivalent of increasing it by 1); then making a payment *immediately* leaves a balance to which r is applied to yield the desired CF each subsequent period. Here,

$$\$100 \times \frac{1}{.07} (1.07) = TOT ;$$

the solution, TOT = \$1,528.57, is \$100 more than with end of year flows. With \$100 repaid immediately, the \$1,428.57 remaining balance would earn 7% interest (\$100) each subsequent year forever.

Example 2: Cash Flow is Unknown

If you borrow \$410.02 and pay 7% interest on the unpaid balance, how much must you pay each year to repay the loan in five years? For year-end payments:

$$PMT \times \frac{1 - \left(\frac{1}{1.07}\right)^5}{.07} = \$410.02 ;$$

PMT = \$100. For *beginning* payments:

$$PMT \times \frac{1 - \left(\frac{1}{1.07}\right)^5}{.07} (1.07) = \$410.02 .$$

The solution, PMT = \$93.46, is *smaller*; if we make the first payment right away, so less interest is owed over the loan's life, we need not pay as much each time. Again, the beauty is that we easily find the answer with PMT x FAC = TOT and some simple algebra.⁸ For a \$410.02 loan with 7% interest and *perpetual* payments:

$$PMT \times \frac{1}{.07} = \$410.02 ,$$

for an *end* of period PMT = \$28.70, or

$$PMT \times \frac{1}{.07} (1.07) = \$410.02 ,$$

for a *beginning* of period PMT of \$26.82.

Example 3: Rate of Return is Unknown

What annual interest rate is the lender charging on a \$410.02 loan with five years of \$100 year-end payments?

$$\$100 \times \frac{1 - \left(\frac{1}{1+r}\right)^5}{r} = \$410.02 ;$$

$r = 7\%$. Again, PMT x FAC = TOT facilitates trial and error. For *beginning* CFs:

$$\$100 \times \frac{1 - \left(\frac{1}{1+r}\right)^5}{r} (1+r) = \$410.02 ;$$

we find $r = 11.034\%$. A \$410.02 balance could provide \$100 annual payments *perpetually* only if the lender earned

$$\$100 \times \frac{1}{r} = \$410.02 ,$$

or $r = 24.389\%$ for *end* of year payments,

$$\text{and } \$100 \times \frac{1}{r} (1+r) = \$410.02 ,$$

or $r = 32.256\%$ for *beginning* payments.

Instructor Notes

Example 4: Time Periods are Unknown
How many year-end payments of \$100 each must you make to retire a \$410.02 loan carrying a 7% annual interest rate?

$$100 \times \frac{1 - \left(\frac{1}{1.07}\right)^n}{.07} = 410.02 ;$$

$n = 5$ years of payments retires the loan.
With *beginning* of year payments:

$$100 \times \frac{1 - \left(\frac{1}{1.07}\right)^n}{.07} (1.07) = 410.02 .$$

If we start today, we have to make only $n = 4.6158$ payments to retire the loan.

More Complex Applications

The versatile $PMT \times FAC = TOT$ equation is equally useful for annuities that change by constant percentages. The factors FAC look much like level annuity factors, except that a rate of expected change g (think growth, though the change could be negative) appears in each formula.

Our FVA factors FAC are

$$\frac{(1+r)^n - (1+g)^n}{r-g}$$

for *end* of period cash flows and

$$\frac{(1+r)^n - (1+g)^n}{r-g} (1+r)$$

for *beginning* of period cash flows.

Say you expect your GNMA mutual fund to provide a 7% average annual return. If your yearly deposits begin at \$100 and increase by 2% (perhaps the expected inflation rate) with each successive period, what will your account be worth by the end of year 5? We solve as

$$100 \times \frac{(1.07)^5 - (1.02)^5}{.07 - .02} = TOT$$

for year-end deposits and

$$100 \times \frac{(1.07)^5 - (1.02)^5}{.07 - .02} (1.07) = TOT$$

for *beginning* deposits. The respective TOT values of \$596.94 and \$638.73 are greater than the \$575.07 and \$615.33 future values in our earlier *level* annuity analysis; you amass more with growing deposits than with a static stream.⁹ (Putting in amounts that *decline* by 2% per period would generate lower TOT values of \$554.03 for *end* of period deposits and \$592.82 for *beginning* deposits.)

Our PVA factors FAC for annuities that change by constant percentages are

$$\frac{1 - \left(\frac{1+g}{1+r}\right)^n}{r-g} \quad \text{and} \quad \frac{1 - \left(\frac{1+g}{1+r}\right)^n}{r-g} (1+r) .$$

Consider simple graduated payments: what is the first payment on a \$410.02 loan with 7% annual interest, to be repaid over five years, if each payment exceeds the previous one by 2%? We solve as:

$$PMT \times \frac{1 - \left(\frac{1.02}{1.07}\right)^5}{.07 - .02} = 410.02 \quad \text{or}$$

$$PMT \times \frac{1 - \left(\frac{1.02}{1.07}\right)^5}{.07 - .02} (1.07) = 410.02 .$$

The \$96.34 and \$90.03 PMT values (*end* and *beginning* of year, respectively) are below the \$100.00 and \$93.46 payments for unchanging five-year streams; the first payment can be lower if subsequent payments rise. (For a *negative* 2% change the initial payment would have to be accordingly *higher* at \$103.80 or \$97.01.)

What if each payment grows by 2%, and the stream continues forever? The PV of a changing perpetuity is the first CF in the series, divided by the required return minus the expected rate of change. Here:

$$PMT \times \frac{1}{.07 - .02} = 410.02 \quad \text{and}$$

$$PMT \times \frac{1}{.07 - .02} (1.07) = 410.02 .$$

The respective first PMT values of \$20.50 and \$19.16 are much lower than the first payments in our five-year growing series.

Other similarities carry through from earlier illustrations. We can solve for any unknown, with all patterns (such as end vs. beginning CFs) shown in level annuity cases applying to changing annuities.¹⁰

Concluding Remarks

$PMT \times FAC = TOT$ is a tool box that helps organize computational tools, a unified framework for identifying/analyzing relationships and solving for unknowns in a range of annuity applications.¹¹ It illuminates structural similarities across annuity examples, while reinforcing the notion that the lump sum ("TOT") is the key to distinguishing between FVAs and PVAs. Because it helps students compute correct answers without frustration, error,

or the need to refer to lengthy example lists or cumbersome if/then diagrams, $PMT \times FAC = TOT$ is an almost foolproof approach to computing annuity values. ■

Notes

- While the technique presented here works nicely with factor tables, students might be encouraged to compute their own factors with the formulas that underlie the table values. Doing so allows more flexibility in picking rates and time periods, facilitates dealing with annuities that change by constant percentages (not doable with typical tables), and makes time value seem less mechanical and mysterious.
- Another way to differentiate the two is that in FVA situations the rate of return is being paid or earned on a *growing* balance, whereas in PVA cases the return is paid or earned on a *declining* balance.
- One way to think about level annuity factors is that the n -period factor for PV of a level *ordinary* annuity is the sum of the PV of \$1 factors for n periods at the same rate, while the n -period factor for FV of a level annuity *due* is the sum of the FV of \$1 factors for n periods. The FVA factor is the corresponding PVA factor multiplied by $(1+r)^n$; this holds true for both level and changing annuities.
- It can be awkward to think of the PV of an annuity due, which would involve an initial cash flow the day the lump sum is paid or received, like borrowing money and immediately making the first payment; or the FV of an ordinary annuity, like opening an IRA but waiting a year to make the first deposit. But legitimate applications exist in both these areas (leasehold valuation is often a PV of annuity due example), and students should be able to analyze these situations.
- We compute the solution in the following steps:

$$\begin{aligned} 100 \times \frac{(1.07)^n - 1}{.07} &= \$575.07 \\ (1.07)^n - 1 &= (.07)(5.7507) = .402549 \\ \frac{(1.07)^n}{1.07} &= 1.402549 \\ n \ln 1.07 &= \ln 1.402549 \\ n(.067659) &= .338291; n = 5 \end{aligned}$$

- Or: to withdraw \$100 from an account in each of the next five years, if you expect a 7% annual return on the remaining balance, how much must you deposit today? Repaying loans and depleting accounts are the same idea, with lender and borrower roles reversed.
- The perpetuity, with its simple attendant computations, can provide a quick estimate of the value of a long-term, but not truly perpetual, annuity; we can typically include or ignore distant future CFs without substantially affecting the total computed PV. The *future* value of a perpetuity is essentially a meaningless concept.
- A common example of a PV of a level annuity with an unknown PMT and a *non-annual* CF stream is the home loan payment. If you borrow \$120,000 and the amortization schedule calls for monthly payments over 30 years (360 total) at a 7.5% stated annual interest rate (an annual percentage rate, or APR; not an effective annual rate, or EAR), the monthly payment is

$$\begin{aligned} PMT \times \frac{1 - \left(\frac{1}{1 + .075/12}\right)^{360}}{.075/12} &= \$120,000 \\ \$120,000 \div 143.0176273 &= PMT, \text{ or} \\ \$120,000 \times .00699215 &= PMT = \$839.06 . \end{aligned}$$

- .00699215 is the reciprocal of the factor for the PV of a level ordinary annuity; it is the "payment factor" by which the loan amount is, by tradition, *multiplied* in computing the payment (just as the *sinking fund* factor is the FVA factor's reciprocal).
- This formula works both when $g < r$ and when $g > r$, but it should ordinarily be that $g < r$, in that growth should be a component of, and therefore something less than, the long-term required rate of return. If $g = 0$ the growing annuity factor simplifies to the factor for the FV of an ordinary level annuity; the latter therefore is a special case of the former (as is also true in the PV of a growing annuity situation).
- CFs in a *declining* percentage perpetuity never reach zero, in theory, but they eventually become so small - tiny fractions of a cent - that it would be impractical to deliver any cash returns. Thus perpetuities with CFs expected to decline in percentage terms raise practicality questions, while a stream declining by a constant *dollar* amount per period could not continue perpetually.
- $PMT \times FAC = TOT$ works equally well with *deferred* annuities (useful in analyzing graduated payment loans, or in valuing properties with step-up rents); one way to compute the factor FAC for deferred annuities with end or beginning of period CFs, respectively, is

$$\frac{1 - \left(\frac{1}{1+r}\right)^n}{r} \left(\frac{1}{1+r}\right)^d \quad \text{or} \quad \frac{1 - \left(\frac{1}{1+r}\right)^n}{r} \left(\frac{1}{1+r}\right)^d (1+r)$$

with n the number of cash flows and d the number of periods before the first cash flow occurs. $PMT \times FAC = TOT$ is also of *some* help in computing values for the type annuity whose CFs change by a constant *dollar* amount D each period, though the analysis is more complex. The equations would be

- $(PMT \times FAC) + D \times [(FAC - n)/r] = TOT$ for the FV case,
- $(PMT \times FAC) + D \times [FAC - n/(1+r)^n]/r = TOT$ for the PV case, and
- $(PMT \times FAC) + (D \times FAC) = TOT$ for the perpetuity case, with FAC in each case representing the corresponding ordinary level annuity factor. All other computation/interpretation patterns shown above hold true in all of these cases, such as adjusting for beginning flows with $(1+r)$ [note that each *term* to the left of the equals sign must be multiplied by $(1+r)$ in the constant dollar change examples].

(continued from page 16)

$$V_{Fee} = PV(MR - OE) + PV(REV) - PV(LE\&FR).$$

Here MR is market-based rent stream, OE is the stream of operating expenses, and LE&FR is the total of lease-up expenses and foregone rent (costs of getting tenants in place). Reversion value REV is the PV, at a hypothetical sale date, of all NOI expected in years *after* that date. That more distant future NOI also reflects market rents, since we are not considering possible effects of long-term leases.

If we do consider the impact of lease contracts in place, however, our focus changes to valuing the *leased fee* estate (the owner's fee interest, subject to the leases), which reflects NOI based on the contract rents. It can be represented as:

$$V_{LF} = PV(CR - OE) + PV(REV).$$

CR is contract rent called for by current leases. V_{LF} is affected by *market* rents only in that contract rents will *become* market rents at various future dates, as expiring leases renew at market levels (affecting both the NOI stream and the reversion).³ Finally, the *leasehold* – the right to occupy the property in return for making contractual rent payments during the lease period – represents the lessee's (tenant's) bundle of rights. The leasehold is worth the PV of the difference between the market and contract rent streams:⁴

$$V_{LH} = PV(MR - CR).$$

Current market rents therefore affect the leasehold value, but not that of the leased fee. If CR is less than current MR, the lessee has negotiated favorable terms, creating a positive leasehold value. If CR exceeds MR, the lessee has committed to paying too much, for a negative leasehold value. The equations can be rearranged to show V_{Fee} equal to V_{LF} plus V_{LH} , minus $PV(LE\&FR)$. V_{LF} thus can be seen as a *residual* equal to unencumbered fee value, plus $PV(LE\&FR)$, minus leasehold value.⁵

Leased Fee as the Property Tax Base

When income real estate changes hands, it is generally the leased fee, not a hypothetical unencumbered fee, that is sold, and for which a price is observed. Should the leased fee value also be the guide for assessing real property taxes? Illinois statutes call for assessing land at its "fair

cash value," most reasonably interpreted as the ownership (fee) interest subject to existing leases (*i.e.*, the leased fee), and some court cases explicitly state that assessors can consider contract rent ("restricted" rent). Even Illinois rulings that let assessors consider *market* (oddly termed "economic") rent do not suggest that current tenants with favorable leases could be evicted by a new owner. Indeed, it is logical to consider market rent only as leases expire, an idea consistent with assessing only the leased fee. In fact, it is clear that assessors do *not* use total real property (unencumbered fee) values as the tax base, since contract rents can be considered in assessments but play no role in valuing an unencumbered fee.

Of course, CR also affects leasehold value, so assessors' use of contract rents *could* be consistent with taxing total property value *if* separate V_{LF} and V_{LH} assessments were made. But it is dubious to think officials would send *negative* tax bills (rebates) to reflect negative leasehold values for unfortunate tenants whose contract rents exceeded market rents. So even though a leasehold is a real property interest, its value figures into real property tax only indirectly, by causing V_{LF} to differ from total real property value.

In other words, taxing only landlords causes a portion of real property value to be omitted from the computation of the *ad valorem* tax base. Positive leasehold values cause leased fees to be worth less than unencumbered fees (if lease-up costs are relatively low), for a tax base lower than total property values; negative leasehold values give a tax base exceeding total real property values.

"Business Enterprise Value"

Is what is thereby ignored in the tax base something other than *real* property value? Some attribute differences between MR and CR-based values as reflecting non-real property *business enterprise value* (BEV) that can be part of the sale price of income real estate, like shopping malls. Arguments purporting to explain how BEV arises all relate to exceptional management effort or skill. Most of them suffer from faulty logic. For example, a common view holds that enhancing the physical environment (providing seating for shoppers' comfort) can create BEV.

The error is the implicit assertion that things like mall seating are not items of real property, a mistake that results from seeing fixtures only as things *literally* affixed (bolted, glued) to land. Other legal tests may apply. Something made specially to use on the premises can be a fixture, and so can any other item, depending (as in our lease example) on intent; an item that is not a fixture in one context can be a fixture in another. Imagine the surprise of a mall buyer whose seller removes the chairs on the day of the closing; the intent surely would be that the furniture is permanent (it stays through the conveyance).

Another argument is that property managers create BEV with correct tenant mixes (stores with different retail cycles give malls insurance from volatility that is not available to owners of freestanding stores), favorable operating agreements (especially with anchor tenants), or effective promotion of a property as a whole (designing good percentage rent schemes, or subsidizing some tenants, to create strategic benefits). Yet these advantages are available to any competent manager, and transfer to new owners (as does the advantage of above-market contract rent gained through exceptional management or fortuitous timing). A shopping mall is *expected* to have furniture, bathrooms, carts, productive lease terms, and a sensible mix of tenants; net returns to these activities are part of the land value. Some feel that new tenants systematically pay lower rents than ongoing tenants because of BEV. Yet a simple alternative explanation is that landlords know how costly it is to move a store⁶ (just as family moving costs compel firms to hire new executives at salaries higher than their peers earn).

BEV is also said to result from monopoly power a shopping mall or signature office building gets from its name recognition. The process is not totally predictable because of a commercial facility's life cycle, but any expected improvement or decline in future reputation should be capitalized into the price paid for the real property. Indeed, it would be impossible *not* to transfer this reputational effect in a sale, just as some part of the value of tenants' identities is impounded into real estate values, since a store or other commercial tenant typically can not change its name without the landlord's approval.

A related argument is that any real estate enterprise benefits from unique proximity to some constituency. There probably is such a *spatial monopoly* advantage, but it would be perceived by buyers and sellers, and impounded into the land value. Of course, the extent of this monopoly power may depend on the public's granting of special *zoning* rights. A seller of unzoned land can not extract all benefits expected from rezoning, but the price should rise by the enhancement to value expected through the new zoning, minus costs of resources to get the zoning changed, since zoning passes in a sale. Another locational argument is that selling tenants utilities at a profit creates BEV. Yet because this ability results from the landlord's control of the point where utilities enter the property, and would transfer in a sale, it is a real property right that should attach to the land value.

So Does BEV Exist?

A well-informed seller thinking of selling land to a shopping mall developer would consider what the most feasible type of facility would generate as a surplus (true "economic rent"), and would try to extract that surplus. Potential competition among developers would increase the chance of success in this quest to capture the expected surplus, from any source, in the land price. In an identical manner, the seller of an existing facility will try to capture the reasonably anticipated future surplus associated with ownership. This relationship reveals the lack of economic logic in most arguments for the existence of substantial non-real property BEV.

Yet there is "business value" that can be separate from assessed real property value. In general, a right whose conveyance would require mention in the sale contract is not an item of real property; its value is BEV in a business context. A restaurant or motel sale would not likely include access to franchise rights, or to existing management, unless noted in the contract, so value attributable to those advantages is not real property value. Licenses that regulators give casinos, bars, or nursing homes provide value through limiting competition, but would not automatically attach to the real property in a sale. Thus, assessors must be cognizant of BEV's existence in some contexts.

Because only V_{LF} is assessed and taxed, what is often incorrectly called "business value" is impounded into V_{LF} because the leases have permanence. Thus, we must dismiss the idea that BEV caused by exceptional management is reason to reduce assessments on income property – if the management advantage is reflected in contract rents.⁷ This outcome is symmetrical, in that atypically *poor* management gives contract rents *below* market rents, thereby reducing V_{LF} and the assessment, to the extent the contracts run into the next owner's tenure.

Eminent domain is different; the basic rule is to compensate the pre- and post-taking value difference. So a tenant should get V_{LH} and a landlord V_{LF} , for a V_{Fee} total. But if V_{LH} is negative, the total could be V_{Fee} only if the tenant actually paid something to the fee owner (impractical, though logical in that the tenant gets out of an unfavorable lease). So the public sector must pay a negative leasehold's absolute value. An unfortunate asymmetry is that for compensation to equal V_{Fee} with positive V_{LH} , the landlord must get V_{LF} and the tenant the positive V_{LH} .

A Numerical Example

Suppose a particular income property is expected to be sold every two years. In each year half of the leases are new, so while a lease also lasts two years, half of the building's leases overlap different owners. Expected future cash flows (net operating income [NOI] and selling price [SP], with rent and expenses paid at the start of each period) are discounted at a 10% annual rate. Expectations are *myopic*; future revenue and expense figures are expected to equal today's, even though today's differ from last year's.

Above-market rent can exist for two reasons that relate to extraordinary property management. Tenants sign long-term leases at above-market rents, expecting the outstanding management service to continue throughout the lease term, and will be disappointed later if less capable management takes over; we call this premium *fixed* excess rent. Rent exceeding market levels only while supernormal management is in place, and not continuing after a sale (think of a percentage of sales, or increments provided by a franchise) is dubbed *variable* excess rent.

Fixed and variable excess rents have profoundly different effects on value.

Consider a building with \$4,000 in market rent and \$2,000 in operating expense expected in every future year. Last year's rent was \$450 above market due to a skilled manager's negotiating higher rents (for effective gross income [EGI] of \$4,450 – \$2,000 = \$2,450), and half that excess continues this year because half the leases existing last year remain (for EGI of \$4,225 – \$2,000 = \$2,225). Thus there is positive fixed, but no variable, excess rent. If management unexpectedly changes, some tenants are stuck paying above-market rent this year without getting above-market service.

V_{LH} today is the negative of this year's \$225 excess rent; last year there was no leasehold value, since high service matched the high rent. V_{LF} (PV of the owner's cash flows) today is \$22,225 (\$2,225 NOI to be received today, plus \$20,000 SP the owner could get today, computed as a \$2,000 future NOI stream capitalized at 10%). Last year, when the owner anticipated selling the property today, V_{LF} was a higher \$22,655 (last year's \$2,450 NOI, plus this year's \$22,225 value discounted for a year). In contrast, V_{Fee} (leased fee plus leasehold value) today is only \$22,000. No BEV is built into the selling price, because no value could be withdrawn by the seller (long-term leases transfer with the land).

If last year's \$450 excess rent was variable, then none of that excess carries forward to today, since the source of that excess would not transfer in a sale. Variable excess rent thus has no influence on SP, so V_{LF} equals V_{Fee} of \$22,000. There is no V_{LH} , since rent adjusts to reflect the expected normal management service. Last year's \$450 excess rent was BEV. ■

Notes

1. A confused jurist might opine that a zoning regulation is not a taking, for example, because the owner still has his "property," even though a smaller bundle of rights remains after this *regulatory* taking.
2. A mortgage lender might think of permanence in the sense of lasting through the term of the loan.
3. Since the property's remaining life at the sale date is likely to be much longer than the remaining terms of leases in existence at that time, we can think of the reversion as being largely a function of expected future *market* rents.
4. The PV of market minus contract rents equals the PV of market rents minus the PV of contract rents; the PV of market rents minus expenses equals the PV of market rents minus the PV of expenses.
5. This relationship is essentially true, though lease-up or absorption periods can affect leasehold value with no offsetting leased fee effect.
6. This argument appears to have been first advanced by University of Wisconsin professor Kerry Vandell.
7. Percentage rent contracts present special problems.

